phases, respectively; C_{1i}, initial concentration in liquid phase at bed input, kg/m³; C'_{1i}, dimensionless concentration in liquid phase at bed input; D_e, mass conductivity, m²/sec; D_{e_0}, initial mass conductivity, m²/sec; D_ℓ, longitudinal diffusion coefficient, m²/sec; k, mass transfer coefficient, m/sec; L, bed length in column, m; x, spatial coordinate in grain, m; R, particle size, m; S, column section, m²; T, parameter taking into account particle shape; V_0, liquid volume in reservoir, m³; v_z, liquid phase velocity in the total column section, m/sec; z, spatial coordinate in bed, m; Z, dimensionless spatial coordinate in bed; β, ratio of the amount of the extracted component in solid phase to flow rate of liquid phase, kg/(m³/sec); ε, bed porosity, m³/m³; ε_1, internal porosity of grain, m³/m³; τ, time, sec; τ', dimensionless time; φ, dimensionless spatial coordinate in grain; ξ, hydromodulus, m³/kg; Bi, Biot number; Pe, Peclet number; w, δ, dimensionless parameters.

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LIQUID AND GAS FILTRATION IN A TWO-LAYER POROUS MATERIAL

V. A. Balashov and N. V. Tyabin

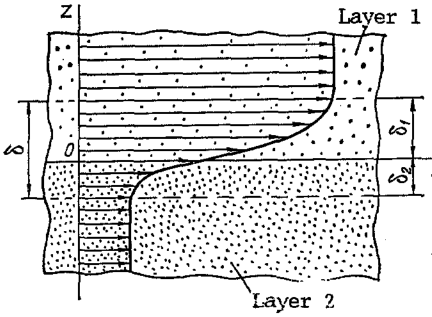
UDC 532.546

A velocity distribution is obtained for a filtration boundary layer near an impermeable wall and near the boundary between two porous layers with differing permeabilities.

Increases in the efficiency of technological processes taking place in porous materials and improvements in apparatus and equipment which use such materials is to a great extent controlled by the need for deeper more detailed studies of the hydrodynamic structure of liquid and gas flows in porous media. One of the unique features of filtration flows which exist in various industrial equipment is that they are often realized under conditions such that the microgeometry of the porous medium, as defined by the size of pores or grainy material particles is comparable to the geometric dimensions of the porous layer itself. In this case, hydrodynamic effects which develop in filtration flow of liquids and gases along the surface of contact of porous materials with another permeable material or an impermeable wall become significant.

Volgograd Polytechnic Institute. Translated from Inzhernerno-Fizicheskii Zhurnal, Vol. 51, No. 4, pp. 542-555, October, 1986. Original article submitted August 12, 1985.

0022-0841/86/5104-1147$12.50



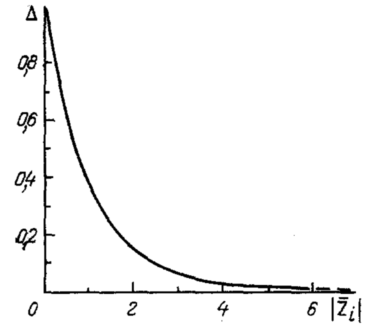


Fig. 1 Fig. 2

Fig. 1. Filtration flow in two-layer porous material.

Fig. 2. Filtration flow parameter vs modulus of dimensionless coordinate.

Theoretical calculations of the motion of an incompressible liquid in a grainy layer with discontinuous change in permeability [1] and an analysis of liquid flow in a tube filled with a granular material having increased permeability near the wall [2] indicate the existence in the region near the boundary between two layers with differing permeabilities of a hydrodynamic boundary filtration layer, within the limits of which smooth merger of the velocity fields of the adjacent filtration flows occurs. Experimental studies have also confirmed the physical reality of such a filtration boundary layer [1]. However, no equations which would permit evaluating the thickness of such a layer or the character of its change are available at present. Such equations would be useful, for example, in solving questions of scale transition in designing heat- and mass-exchange equipment in the chemical and power industries, for analyzing the effect of hydrodynamics on the course of catalytic processes in reactors with a grainy layer, for calculation of special layered filters, and a number of other instances.

We will study filtration flow in a two-layer porous material. We consider steady-state filtration of an incompressible liquid along the extended planar boundary of two layers with differing permeabilities. Within the limits of each layer, let the porous structure of the layer be isotropic, with a discontinuous change on the boundary, while the first layer has the higher permittivity, $k_1 > k_2$.

We introduce a rectangular coordinate system $z-x$, as shown in Fig. 1. The x axis lies on the boundary between the layers and is directed along the filtration flow, while the z axis is directed such that for the first layer $z \geq 0$, and for the second, $z \leq 0$.

At a sufficient distance from the boundary, the filtration velocity in each layer is defined by Darcy's law as

$$v_{xi} = -\frac{k_i}{\mu}\frac{\partial p}{\partial x}, \tag{1}$$

where i = 1, 2 indicate the number of the layer.

Momentum exchange between the filtration flows leads to braking of the flow in the more permeable layer and acceleration of the flow in the layer with lower permeability, as a result of which, within the limits of the boundary zone one can expect smooth merger of the velocity fields of the flows in the neighboring layers, so the velocity field of he filtration flow will appear as shown in Fig. 1. The analysis of the hydrodynamic situation in the boundary region reduces to study of a plane-parallel filtration flow $v_{xi} = v_{xi}(z)$, for which we obtain from the known equations of motion of a filtration flow [3, 4]

$$\frac{\partial p}{\partial x} + \frac{\mu}{k_i} v_{xi} - \mu_{fi}\frac{\partial^2 v_{xi}}{\partial z^2} = 0. \tag{2}$$

With sufficient removal from the boundary between the layers the filtration velocity is equal to that given by Darcy's law, Eq. (1), and we can then say that

$$\frac{\partial v_{x1}}{\partial z} = 0 \text{ as } z \to \infty \text{ and } \frac{\partial v_{x2}}{\partial z} = 0 \text{ as } z \to -\infty. \tag{3}$$

Upon transition through the boundary the average filtration velocity must be continuous and tangent stresses in the flow must be equal, so for z = 0 we write

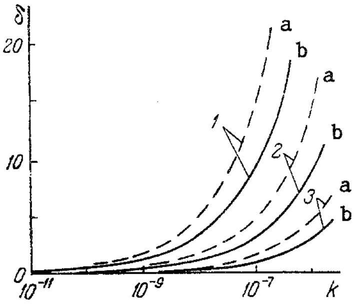


Fig. 3. Filtration boundary-layer thickness δ (mm) vs permeability k (m^2) for various ratios of apparent and physical viscosities of liquid being filtered: 1) $\mu_f/\mu = 10^2$; 2) 10; 3) 1 (a – $\Delta^* = 0.01$; b – 0.05).

$$v_{x1} = v_{x2} \text{ and } \mu_{f1} \frac{\partial v_{x1}}{\partial z} = \mu_{f2} \frac{\partial v_{x2}}{\partial z}. \tag{4}$$

Introducing dimensionless velocity and coordinate with the expressions

$$\bar{v}_{xi} = \frac{v_{xi}}{-\frac{k_i}{\mu}\frac{\partial p}{\partial x}} \text{ and } \bar{z}_i = \frac{z}{\sqrt{k_i \frac{\mu_{\phi i}}{\mu}}},$$

where $z \geq 0$ for $i = 1$ and $z \leq 0$ for $i = 2$, then solving Eq. (2) with conditions (3), (4), we obtain an equation for determination of the filtration velocity in the neighboring layers, which can be written in dimensionless form as

$$\bar{v}_{xi} = 1 + a_i \exp[(-1)^i \bar{z}_i], \tag{5}$$

where

$$a_i = \frac{(-1)^i (k_1 - k_2)}{k_i \left[1 + \sqrt{\left(\frac{k_1}{k_2}\frac{\mu_{f2}}{\mu_{f1}}\right)^{(-1)^i}}\right]}.$$

Because of the condition of equality of the average filtration velocities on the boundary, the condition

$$\frac{\bar{v}_{x1}}{\bar{v}_{x2}} = \frac{k_2}{k_1},$$

must be satisfied, while with removal from the boundary the magnitude of the dimensionless filtration velocity quite rapidly approaches unity.

According to Eq. (5), we may write

$$\Delta = \exp[(-1)^i \bar{z}_i], \tag{6}$$

where $\Delta = (\bar{v}_{xi} - 1)/a_i$. We term the quantity Δ the filtration flow parameter. Figure 2 shows a graph of the function $\Delta = f(|\bar{z}_i|)$ constructed on the basis of Eq. (6). On the boundary between the adjacent filtration flows $\Delta = 1$, and with removal from the boundary the value of the filtration flow parameter decreases exponentially, approaching zero. We may arbitrarily take for the filtration boundary layer thickness the value $\delta_i = |z|$ at which the value of the filtration flow parameter becomes sufficiently small and equal to Δ^*, then on the basis of Eq. (6)

$$\delta_i = (-1)^{-1} \sqrt{k_i \frac{\mu_{fi}}{\mu}} \ln \Delta^*. \tag{7}$$

Figure 3 shows the dependence of filtration boundary-layer thickness calculated by Eq. (7) for two values of the filtration flow parameter, 0.05 and 0.01. It is evident from the figure that for filtration flows in slightly permeable porous materials the thickness of the filtration boundary layer is relatively small, while for materials with high permeability, the thickness of this layer increases significantly, growing with increase in the ratio of the flow filtration viscosity to the physical viscosity of the liquid.

At low permeabilities of the second layer, where $k_1 \gg k_2$, the filtration velocity in the latter is negligibly small, and it can be assumed that for the region with low permeability and on the boundary $v_{x2} = 0$. Such a filtration flow is in fact a flow of a liquid in a porous material along an impermeable boundary to which the liquid adheres. For these conditions on the basis of Eqs. (5) and (7), assuming that $\bar{v}_{x1} = \bar{v}_x$, $k_1 = k$, $\mu_{f1} = \mu_f$; $\bar{z}_1 = \bar{z}$ and $\delta_1 = \delta$, we obtain

$$\bar{v}_x = 1 - \exp(-\bar{z}), \tag{8}$$

$$\delta = -\sqrt{k \frac{\mu_f}{\mu}} \ln \Delta^*. \tag{9}$$

Equation (8) may also be obtained by direct solution of equation of motion (2) for filtration along an impermeable planar wall with liquid adhesion thereon. Equations (8) and (9) can be used to calculate filtration velocity along an impermeable wall and the thickness of the filtration boundary layer only if the permeability of the material is constant down to the boundary.

It should be noted that at present no reliable recommendations are available for determining the apparent velocity of the filtering liquid and Eqs. (7) and (9) can be used to evaluate the quantity μ_f from experimental data determining the thickness of the filtration boundary layer.

It is obvious that if the dimensions of the filtration boundary layer are small enough, it can be neglected in calculating volume flow rates in porous materials of large thickness, assuming the filtration velocity to be a continuous function near the impermeable wall and on the contact boundary between two layers of differing permeability. However, for filtration in thin layers of porous material, the thickness of which may be comparable to the dimensions of the filtration boundary layer, the presence of this layer must be considered in the calculations.

Thus, when performing calculations of concrete filtration flows the thickness of the filtration boundary layer must be evaluated. The principles obtained herein permit such an evaluation, and moreover, can be useful in studying the physical pattern of liquid and gas flow in multilayer porous media and in hydrodynamic modeling of equipment utilizing porous materials.

NOTATION

x, z, Cartesian coordinates; $\bar{z}$, dimensionless coordinate; i = 1, 2, porous layer numbers; v_x, filtration velocity; $\bar{v}_x$, dimensionless filtration velocity; p, pressure; k, permeability; μ, physical viscosity; μ_f, apparent viscosity of filtering liquid; δ, thickness of filtration boundary layer.

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